Sparse Reflections Analysis (SRA)

Fast Removal of General Multipath for Time of Flight Sensors

Eyal Krupka
Microsoft Research Israel

Joint work with: Daniel Freedman, Yoni Smolin, Ido Leichter, Mirko Schmidt
Microsoft Research Israel (ATL)

• Hand pose recognition

• Face recognition

• Depth camera signal processing
Introduction: Time-of-Flight Camera

• A time-of-flight sensor *sends out light* and measures how long it takes *until it comes back*.

• Resolution of 1.5cm, requires sensitivity of $10^{-12}$ seconds!
• 100,000s of pixels per frame
Phase Measurements ToF

Example:

\[ \lambda = 1.5 \text{m}, D = 100 \text{cm}, \phi = 240^\circ \]
\[ D = 101 \text{cm}, \phi = 242.4^\circ \]
\[ D = 250 \text{cm}, \phi = 240^\circ \]

Solved by using multiple (3) modulation frequencies
Multipath (two path case)

Direct path - shortest
Camera -> Floor -> Camera

Indirect path
Camera -> Wall -> Floor -> Camera

\[ v_1 = A_1 e^{2\pi j \frac{D_1}{\lambda_1}} + A_2 e^{2\pi j \frac{D_2}{\lambda_1}} \]

\[ D_2 = D_1 + D_e + D_1 \]
What is Diffuse Multipath?

- A Lambertian surface gives off an infinitesimal amount of light in all directions.

- Multipath results from adding up an infinite number of paths.
Multipath Characterization: Backscattering

• Simple (e.g. two-path) multipath is characterized by a delta-response.

• Diffuse multipath is characterized by a smooth response.

• Combinations are possible: two-path, three-path, two-path + diffuse, etc.
Multipath & depth errors

Reflections & missing floors

Melting into the wall
Multipath Characterization: Backscattering

- Learning the class of backscatterings from a simulator is hard.

- Instead: The backscatterings are a subset of “compressible signals”.

\[ x_{I(t)} \leq Ri^{-1/r} \quad \text{with} \quad r \leq 1 \]

- A generalization of sparse signals.
Our Goal: Deal with Multipath

• Multipath causes major problems in depth estimation. So diagnose and hopefully eliminate it.

• We want to take into account many kinds of multipath, not just the “simple” kind.

• Need a lightweight algorithm.
  • Solve depth & multipath for over 5,000,000 pixels per second! (processing time <0.2 μS per pixel)
SRA: Sparse Reflections Analysis

• For backscattering \( x \), the 3D measurement is
  \[
  v_k = \sum_{j=1}^{n} x_j e^{\frac{2\pi i D_j}{\lambda_k}}
  \]  
  (Phase Modulation ToF)

• Matrix-vector form:

\[
\begin{bmatrix}
  v_1 \\
v_2 \\
v_3
\end{bmatrix}
= \begin{bmatrix}
  2\pi i D_1 \\
e^{\frac{\lambda_1}{\lambda}} \\
2\pi i D_1 \\
e^{\frac{\lambda_2}{\lambda}} \\
2\pi i D_1 \\
e^{\frac{\lambda_3}{\lambda}} \\
\vdots
\end{bmatrix}
\begin{bmatrix}
x_1 \\
\vdots \\
x_n
\end{bmatrix}
\]

the return from \( D_j \) alone

Two-path example:
140 cm and 230 cm.
(Here \( D_j \) is \( j \) cm.)

\( x_j \) is sparse: only 2 non-zero elements.
SRA: Sparse Reflections Analysis

- x is compressible, so a good way of recovering x is to solve
  \[ \min_x \|x\|_0 \quad \text{subject to} \quad v = \Phi x \quad \text{and} \quad x \geq 0 \]

- The above is NP-hard. The following convex program provably yields the same solution:
  \[ \min_x \|x\|_1 \quad \text{subject to} \quad v = \Phi x \quad \text{and} \quad x \geq 0 \]

- In practice, we convert everything from complex to real.
SRA: Sparse Reflections Analysis

• There is a standard trick for converting an $L_1$ optimization to a linear program (LP).

• In our case, it is easier. Since $x \geq 0$, we have that

$$\|x\|_1 = \sum_{i=1}^{n} |x_i| = \sum_{i=1}^{n} x_i = 1^T x.$$ 

• Thus, we only have to solve the LP:

$$\min_{x} 1^T x \quad \text{subject to} \quad \nu = \Phi x \quad \text{and} \quad x \geq 0$$

We can use standard solvers (e.g. Matlab).
Accounting for Noise

• It is possible to add in Gaussian noise considerations to the convex program directly.

\[
\min_{x} 1^T x \quad \text{subject to} \quad \|\Phi x - v\|_2 \leq \epsilon \|v\|_2 \quad \text{and} \quad x \geq 0
\]

• To simplify, we substitute an L\(_1\) norm for the L\(_2\) norm:

\[
\min_{x} 1^T x \quad \text{subject to} \quad \|\Phi x - v\|_1 \leq \epsilon \|v\|_1 \quad \text{and} \quad x \geq 0
\]

• Now we have an LP again.
Accounting for Noise

• Even with this step, it is still possible to generate *spurious peaks*, which are *solely due to noise*.

• This is due to the parameter $\epsilon$.

• **Additional filtering step:**
  Given an approximate *noise model*, only keep those peaks which are within the signal regime, i.e. *above the noise regime*. 
Comments on the Algorithm

• Leads to a convex program for the depth. Global optimum!

• Is sufficiently fast for offline use. (10 ms) And an even faster approximation algorithm. (1 ms)
  For real time processing our budget is only \( \sim 0.1 \mu S \)

• Tends to find a sparse rather than compressible solution.
  • Solution 1: Can include “diffuse pieces” as atoms in the sparse decomposition (instead of just \( \delta \)’s).
  • Solution 2: If the diffuse piece is narrow (support less than 20 cm), then sparse yields a sufficiently good approximation.
Making it Real-Time

- 6D lookup table requires $O(30\text{GB})$ memory
  - Our budget is $O(10\text{MB})$

- We use a novel 6D to 4D transformation.

- 6D to 5D is easy: $\mathbf{v} \rightarrow \mathbf{v}/\|\mathbf{v}\|$ (either 1- or 2-norm).

- 5D to 4D: $\text{ALG}(\mathbf{v})$ gives the same result as $\text{ALG}(f_\Delta \odot \mathbf{v}) + \Delta$, where
  \[
  f_\Delta = \begin{bmatrix}
  e^{-2\pi i\Delta/\lambda_1}, e^{-2\pi i\Delta/\lambda_2}, e^{-2\pi i\Delta/\lambda_3}
  \end{bmatrix}
  \]

- Choose $\Delta$ “canonically”, e.g., to make first phase zero.
Making it Real-Time

- Idea: use our method to build a 4D Look-Up Table.
Experiments: Two-Path

- Two-path is still a very important case!

Example - SNR = 5, MP strength=5
For SRA, error is \(~4\)cm
For Godbaz, error is \(~100\)cm
Experiments: Two-Path

![Mean Absolute Error SRA](image)

SRA
Example 1

IR

Depth

No Correction

With Correction
Example 2

IR

Depth

No Correction

With Correction
More ways to reduce multipath

• SRA finds a single solution

• For a very low SNR:
  • Alternative solutions may be hidden

• Machine learning solution (Bayesian Risk Minimization)

Generate training set (100,000,000 random multipath patterns)

Environment & multipath simulation → Camera simulation → Machine learning: Bayesian Risk Minimization
More ways to reduce multipath

• SRA finds a single solution

• Learning / Bayesian Risk Minimization for a very low SNR

• Design of modulation frequencies
Example 3

Direct solution

Multipath correction
Summary

• We have tackled \textit{multipath} and depth estimation ...

• using a \textit{theoretically innovative} approach ...

• while achieving \textit{practical, real-time} performance.