# Designing Efficient Map-Reduce Algorithms

Review of Map-Reduce
A Common Mistake
Size/Communication Trade-Off
Specific Tradeoffs

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# Review of Map-Reduce

Mappers and Reducers
Key-Value Pairs
Example Application: Join

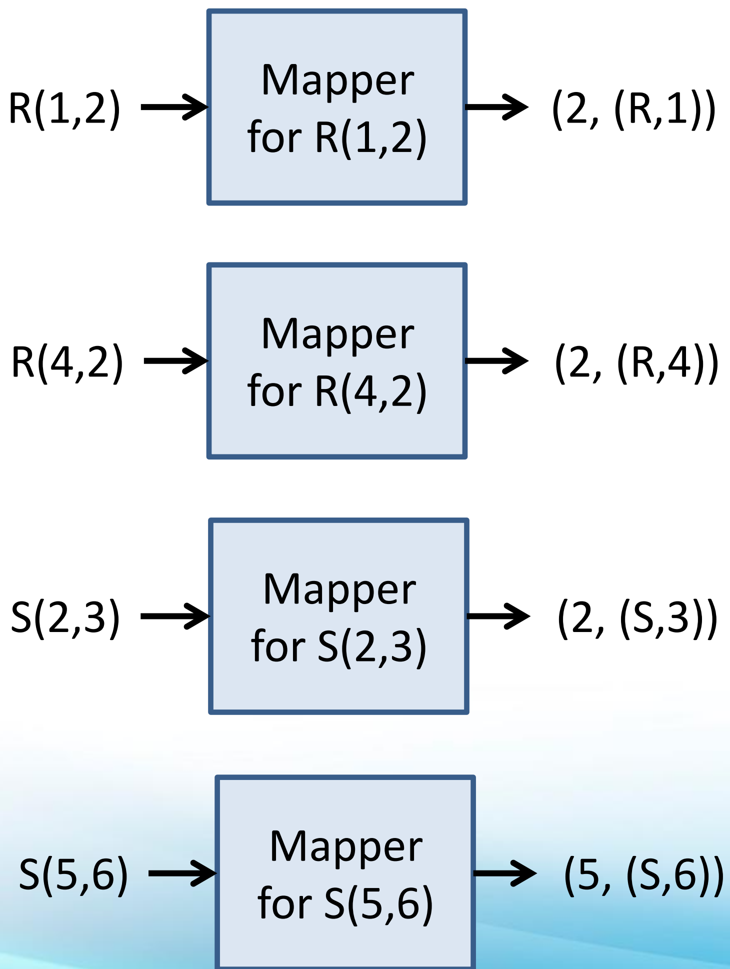
# Mappers and Reducers

- Map-Reduce job =
  - Map function (inputs -> key-value pairs) +
  - Reduce function (key and list of values -> outputs).
- Map and Reduce Tasks apply Map or Reduce function to (typically) many of their inputs.
  - Unit of parallelism.
- *Mapper* = application of the Map function to a single input.
- *Reducer* = application of the Reduce function to a single key-(list of values) pair.

# Example: Natural Join

- Join of R(A,B) with S(B,C) is the set of tuples (a,b,c) such that (a,b) is in R and (b,c) is in S.
- Mappers need to send R(a,b) and S(b,c) to the same reducer, so they can be joined there.
- Mapper output: key = B-value, value = relation and other component (A or C).
  - Example: R(1,2) -> (2, (R,1))
    S(2,3) -> (2, (S,3))

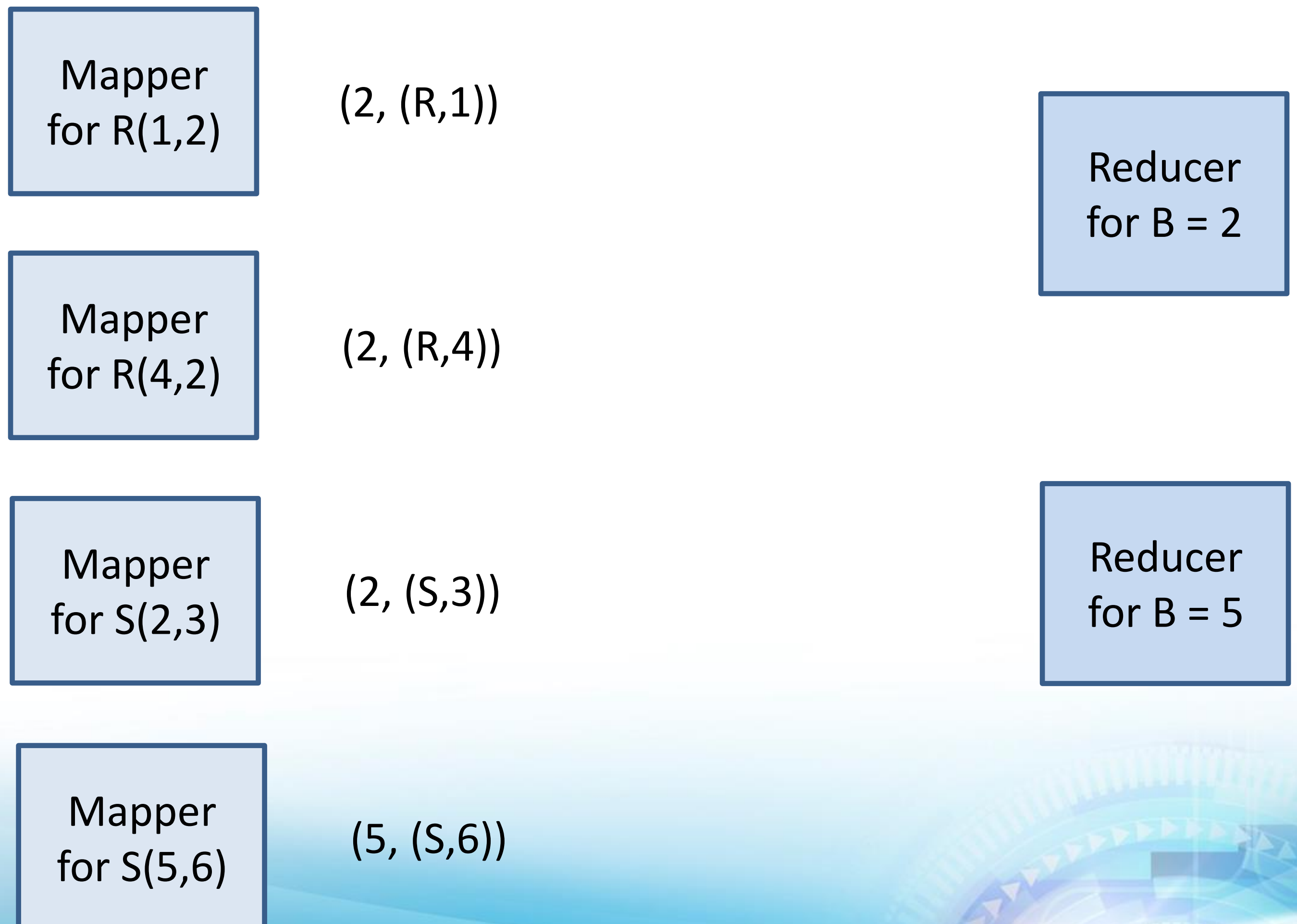
# Mapping Tuples



# Grouping Phase

- There is a reducer for each key.
- Every key-value pair generated by any mapper is sent to the reducer for its key.

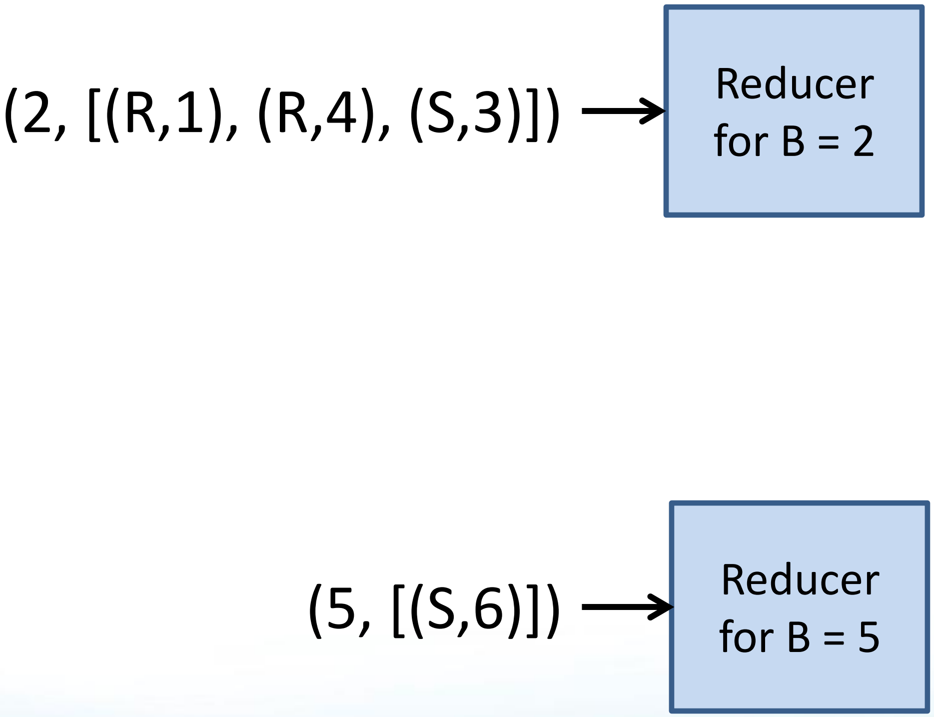
# Mapping Tuples



# Constructing Value-Lists

- The input to each reducer is organized by the system into a pair:
  - The key.
  - The list of values associated with that key.

# The Value-List Format



# The Reduce Function for Join

- Given key *b* and a list of values that are either $(R, a_i)$ or $(S, c_j)$, output each triple $(a_i, b, c_j)$.
  - Thus, the number of outputs made by a reducer is the product of the number of R's on the list and the number of S's on the list.

# Output of the Reducers

(2, [(R,1), (R,4), (S,3)]) → Reducer for B = 2 → (1,2,3), (4,2,3)

(5, [(S,6)]) → Reducer for B = 5 →

# Motivating Example

The Drug Interaction Problem
A Failed Attempt
Lowering the Communication

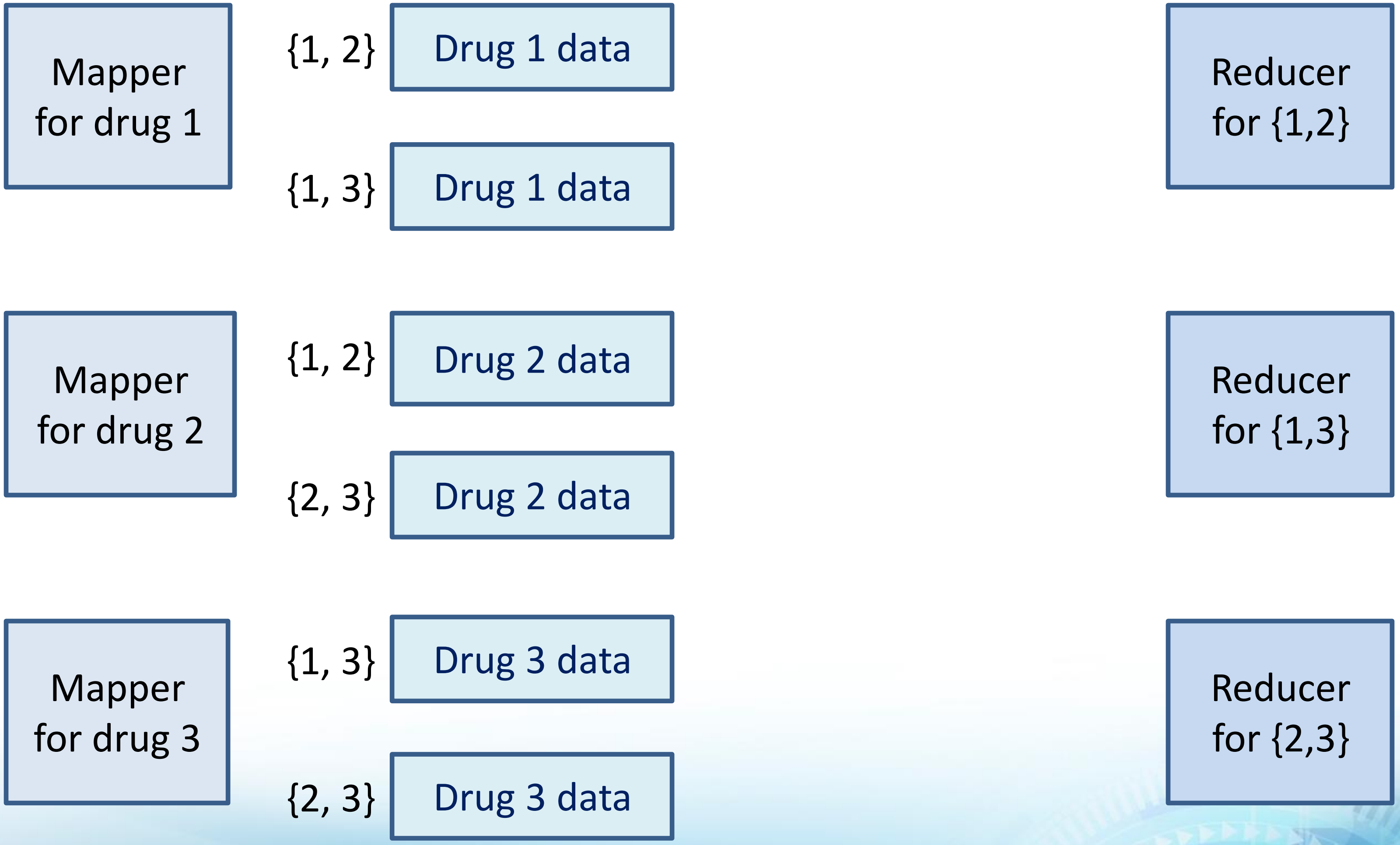
# The Drug-Interaction Problem

- Data consists of records for 3000 drugs.
  - List of patients taking, dates, diagnoses.
  - About 1M of data per drug.
- Problem is to find drug interactions.
  - Example: two drugs that when taken together increase the risk of heart attack.
- Must examine each pair of drugs and compare their data.

# Initial Map-Reduce Algorithm

- The first attempt used the following plan:
  - Key = set of two drugs {*i*, *j*}.
  - Value = the record for one of these drugs.
- Given drug *i* and its record $R_i$, the mapper generates all key-value pairs ({*i*, *j*}, $R_i$), where *j* is any other drug besides *i*.
- Each reducer receives its key and a list of the two records for that pair: ({*i*, *j*}, [$R_i$, $R_j$]).

# Example: Three Drugs



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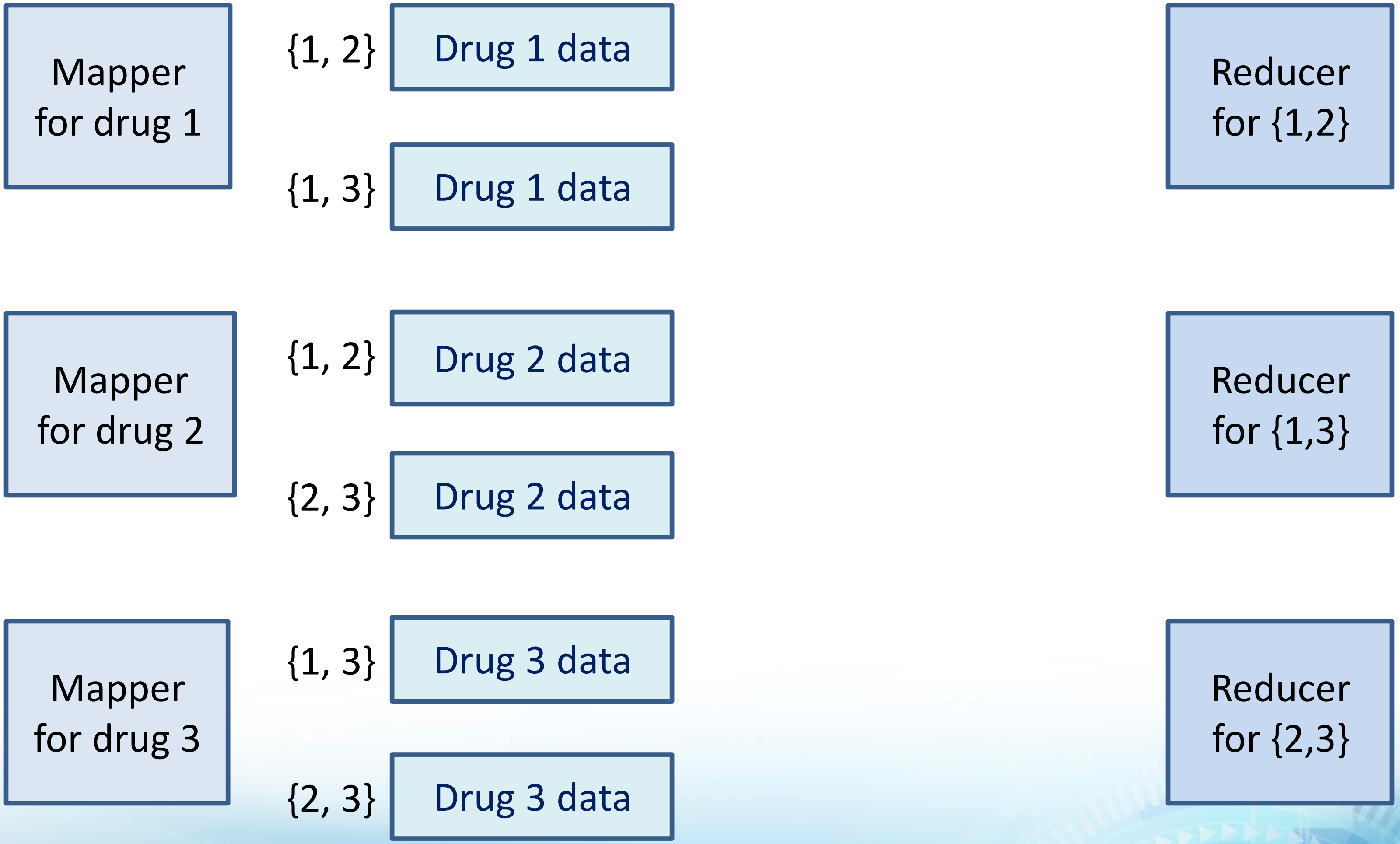
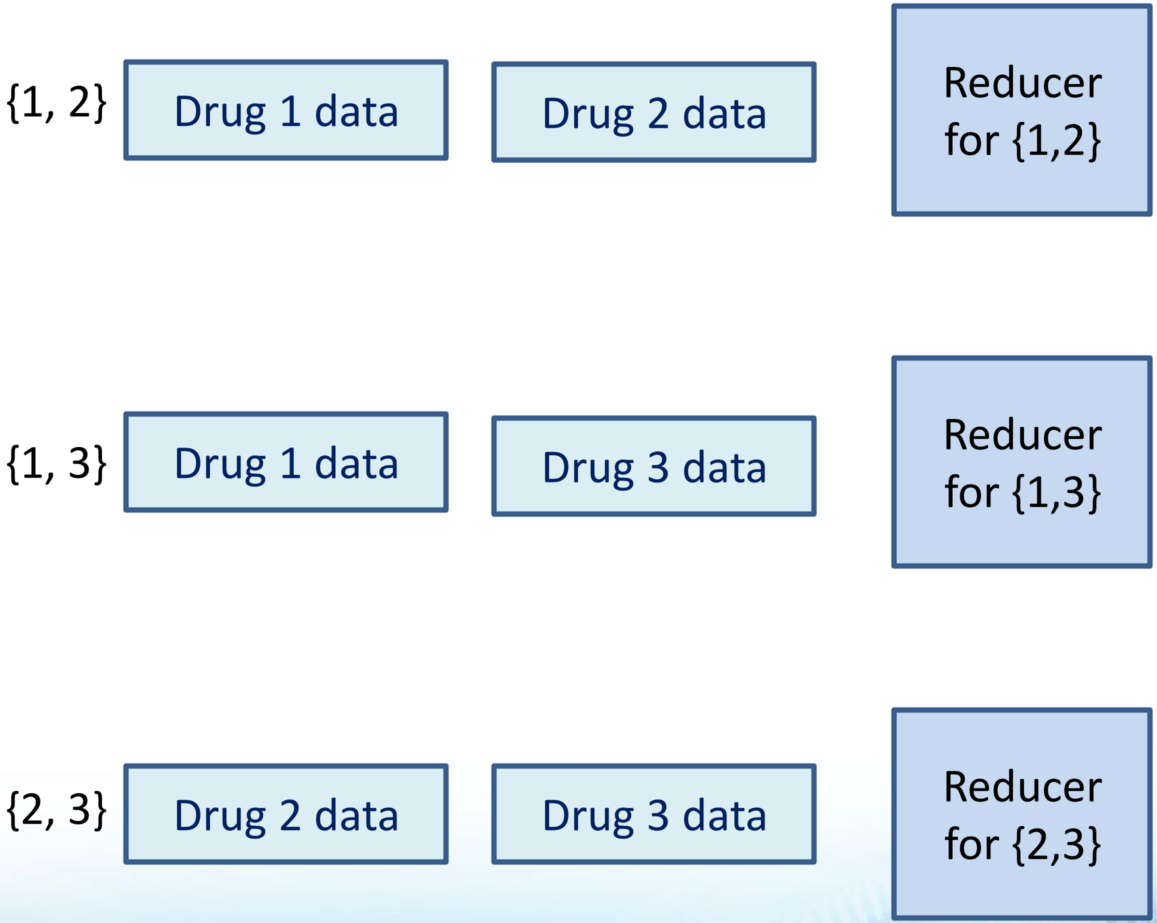


# Example: Three Drugs



# What Went Wrong?

- 3000 drugs
- times 2999 key-value pairs per drug
- times 1,000,000 bytes per key-value pair
- = 9 terabytes communicated over a 1Gb Ethernet
- = 90,000 seconds of network use.

# A Better Approach

- Suppose we group the drugs into 30 groups of 100 drugs each.
  - Say $G_1$ = drugs 1-100, $G_2$ = drugs 101-200,…, $G_{30}$ = drugs 2901-3000.
  - Let g(i) = the number of the group into which drug *i* goes.

# The Map Function

- A key is a set of two group numbers.
- The mapper for drug *i* produces 29 key-value pairs.
  - Each key is the set containing g(i) and one of the other group numbers.
  - The value is a pair consisting of the drug number i and the megabyte-long record for drug *i*.

# The Reduce Function

- The reducer for pair of groups $\{m, n\}$ gets that key and a list of 200 drug records – the drugs belonging to groups $m$ and $n$.
- Its job is to compare each record from group $m$ with each record from group $n$.
  - Special case: also compare records in group $n$ with each other, if $m = n+1$ or if $n = 30$ and $m = 1$.
- Notice each pair of records is compared at exactly one reducer, so the total computation is not increased.

# The New Communication Cost

- The big difference is in the communication requirement.
- Now, each of 3000 drugs’ 1MB records is replicated 29 times.
  - Communication cost = 87GB, vs. 9TB.

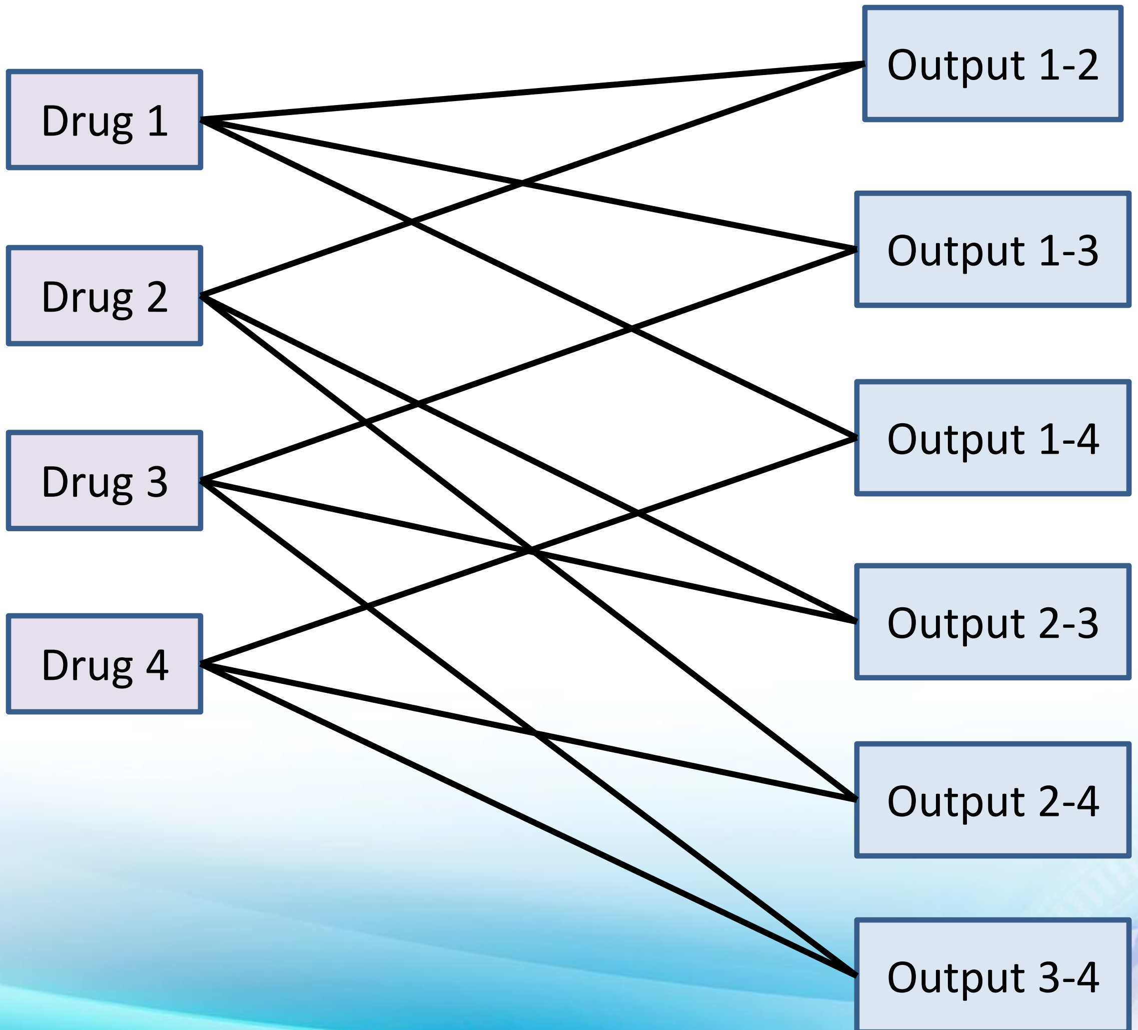
# Theory of Map-Reduce Algorithms

Reducer Size
Replication Rate
Mapping Schemas
Lower Bounds

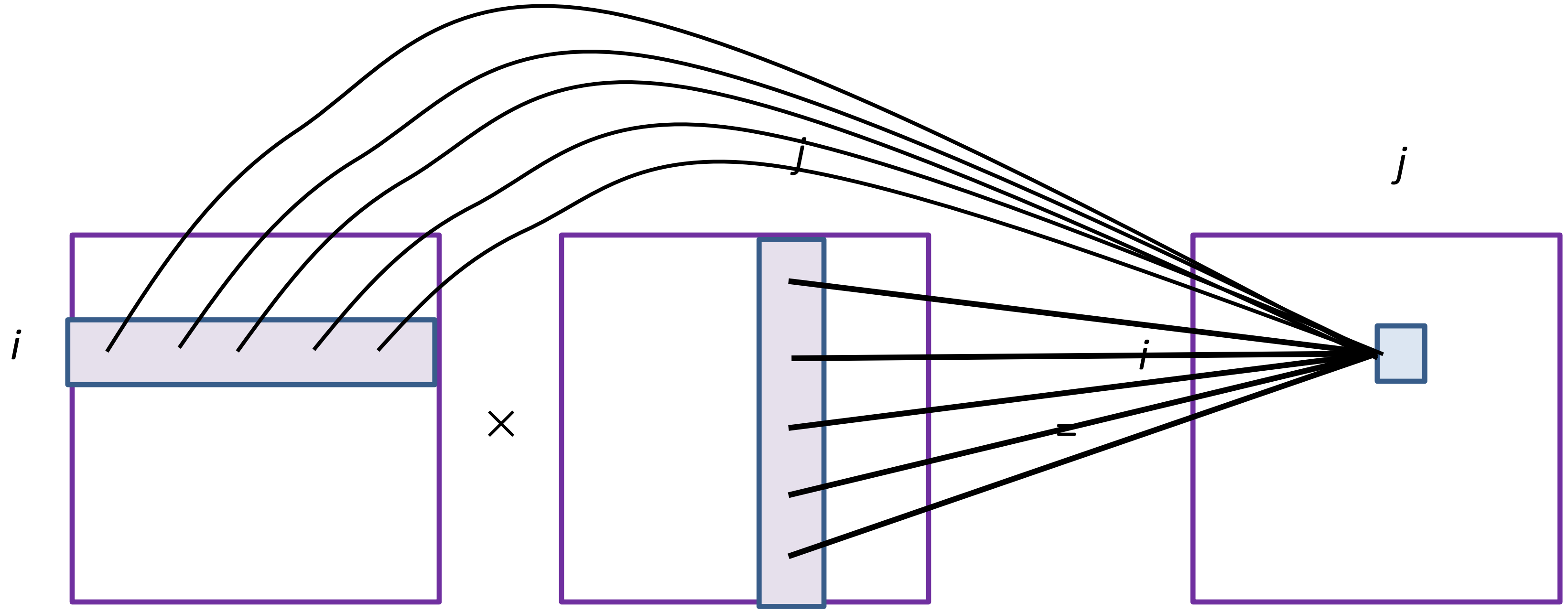
# A Model for Map-Reduce Algorithms

1. A set of *inputs*.
   – Example: the drug records.
2. A set of *outputs*.
   – Example: One output for each pair of drugs.
3. A many-many relationship between each output and the inputs needed to compute it.
   – Example: The output for the pair of drugs {*i*, *j*} is related to inputs *i* and *j*.

# Example: Drug Inputs/Outputs



# Example: Matrix Multiplication



# Reducer Size

- *Reducer size*, denoted q, is the maximum number of inputs that a given reducer can have.
  - I.e., the length of the value list.
- Limit might be based on how many inputs can be handled in main memory.
- Or: make q low to force lots of parallelism.

# Replication Rate

- The average number of key-value pairs created by each mapper is the *replication rate*.
  - Denoted r.
- Represents the communication cost per input.

# Example: Drug Interaction

- Suppose we use g groups and d drugs.
- A reducer needs two groups, so $q = 2d/g$.
- Each of the d inputs is sent to g-1 reducers, or approximately $r = g$.
- Replace g by $r$ in $q = 2d/g$ to get $r = 2d/q$.

Tradeoff!
The bigger the reducers,
the less communication.

# Upper and Lower Bounds on r

- What we did gives an upper bound on r as a function of q.
- A solid investigation of map-reduce algorithms for a problem includes lower bounds.
  - Proofs that you cannot have lower r for a given q.

# Proofs Need Mapping Schemas

- A *mapping schema* for a problem and a reducer size q is an assignment of inputs to sets of reducers, with two conditions:
  1. No reducer is assigned more than q inputs.
  2. For every output, there is some reducer that receives all of the inputs associated with that output.
     - Say the reducer *covers* the output.

# Mapping Schemas – (2)

- Every map-reduce algorithm has a mapping schema.
- The requirement that there be a mapping schema is what distinguishes map-reduce algorithms from general parallel algorithms.

# Example: Drug Interactions

- d drugs, reducer size $q$.
- No reducer can cover more than $q^2/2$ outputs.
- There are $d^2/2$ outputs that must be covered.
- Therefore, we need at least $d^2/q^2$ reducers.
- Each reducer gets $q$ inputs, so replication $r$ is at least $q(d^2/q^2)/d = d/q$.
- Half the $r$ from the algorithm we described.

Inputs per reducer

Number of reducers

Divided by number of inputs

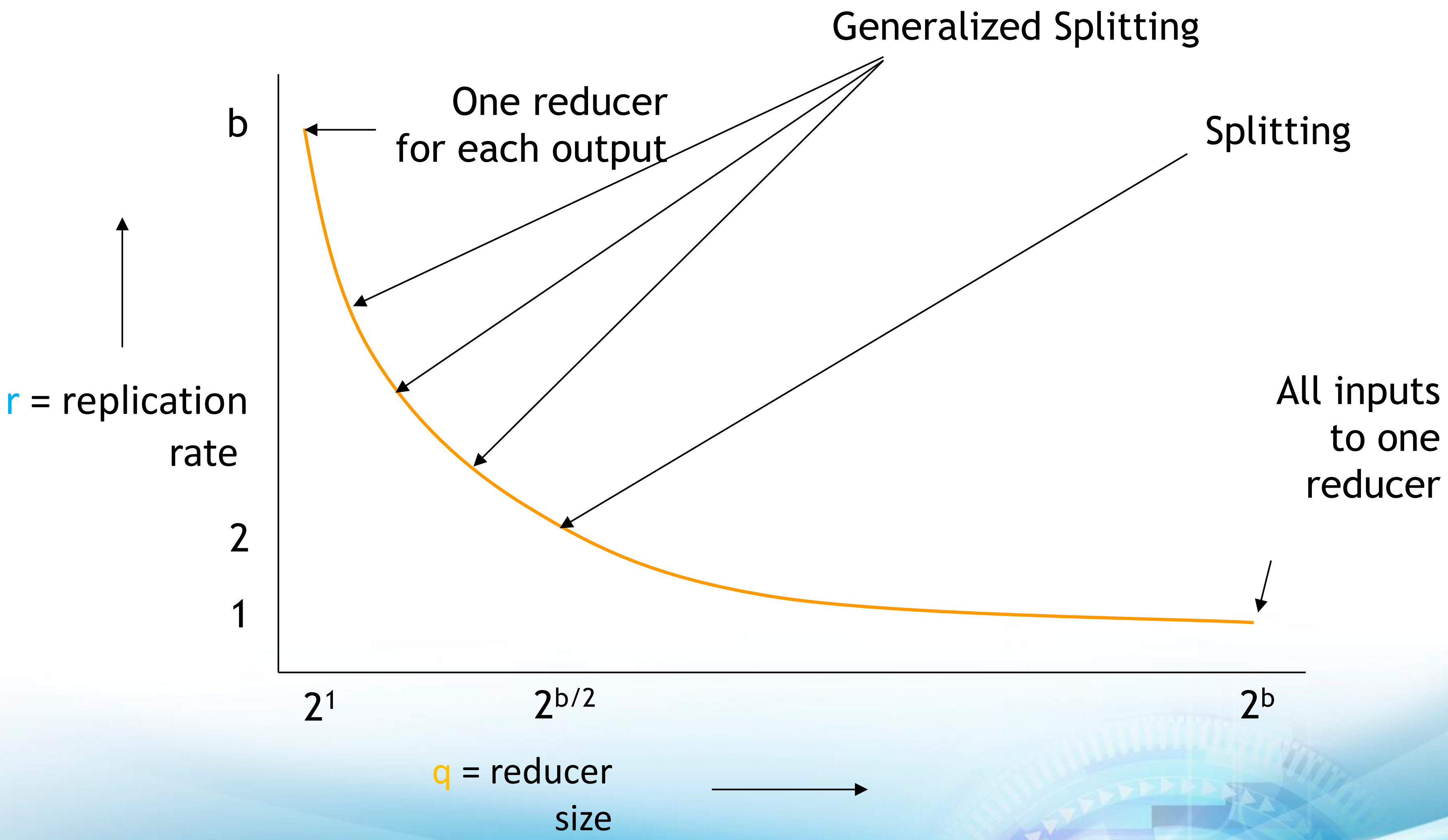
# Specific Problems

Hamming Distance 1
Matrix Multiplication

# Definition of HD1 Problem

- Given a set of bit strings of length b, find all those that differ in exactly one bit.
- Theorem: $r \geq b/\log_2 q$.

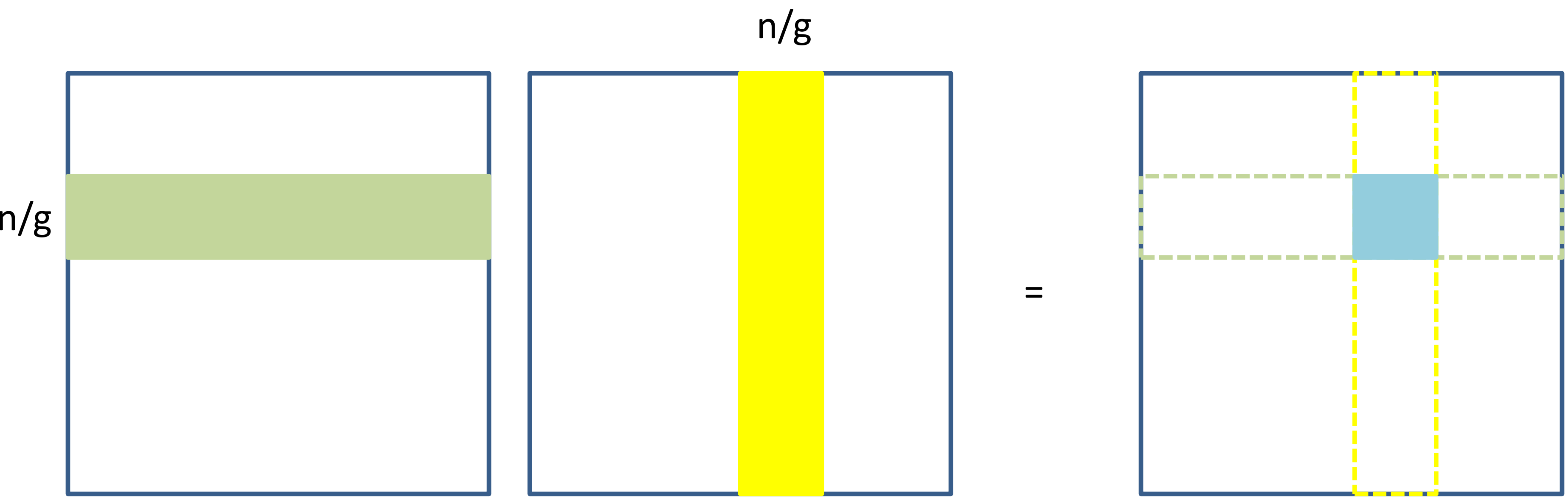
# Algorithms Matching Lower Bound



# Matrix Multiplication

- Assume $n \times n$ matrices $AB = C$.
- Theorem: For matrix multiplication, $r \geq 2n^2/q$.

# Matching Algorithm



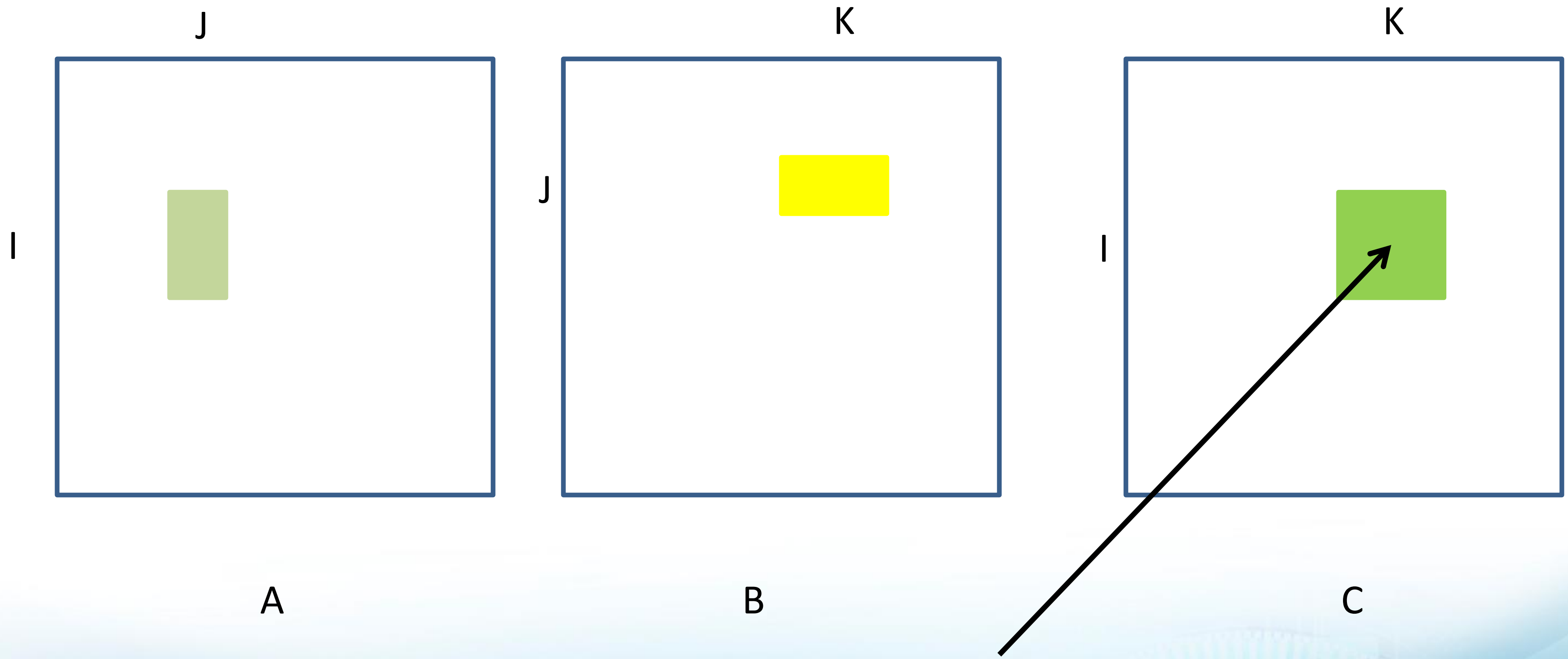

Divide rows of A and columns of B into g groups gives

$r = g = 2n^2/q$

# Two-Job Map-Reduce Algorithm

- A better way: use two map-reduce jobs.
- Job 1: Divide both input matrices into rectangles.
  - Reducer takes two rectangles and produces partial sums of certain outputs.
- Job 2: Sum the partial sums.

# Picture of First Job




For $i$ in I and $k$ in K, contribution is $\Sigma_{j \text{ in } J} A_{ij} \times B_{jk}$

# Comparison: Communication Cost

- One-job method: Total communication = $4n^4/q$.
- Two-job method Total communication = $4n^3/\sqrt{q}$.
  - Since $q < n^2$ (or we really have a serial implementation), two jobs wins!

# Summary

- Represent problems by mapping schemas
- Get upper bounds on number of covered outputs as a function of reducer size.
- Turn these into lower bounds on replication rate as a function of reducer size.
- For HD = 1 problem: exact match between upper and lower bounds.
- 1-job matrix multiplication analyzed exactly.
- But 2-job MM yields better total communication.